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Low-cost method of calculation in investigation accident by random number generation

GUSTAVO A. ENCISO *EVU*

Abstract

With the development of computer science in the last century, the traffic accident reconstruction by "forward" methods (positive time step) [1], it has been growing to find the computer tools that make easy the collision solutions when the data (from evidence) are limited or scarce.

Latina America's experts have not yet used the "forward" methods to solving traffic accident collisions. The "backward" method (negative time step) is the most used to resolve the investigations about car velocities involved in traffic collisions. In frequent opportunities, the uncertainty of the variables does not allow us to know the right solutions.

This work proposal discusses a low-cost method called MERNG - minimization error by random number generation [2][3] to find the initial collision velocity used in classic physics models when the values of variables (evidence) cannot be specified and unknown their probabilities distributions.

Through the random numbers generation, it is possible to find the condition of the variables that allow minimizing the error between the results calculated by MERNG method and the actual distance from the rest position of the cars to the impact area.

This method could be a better alternative to the problem of uncertainty in deterministic models [4], even when the number of variables in the mathematical model is not small.

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CORRESPONDING AUTHOR: PROF GUSTAVO A. ENCISO, GENCISO@CEIRAT.COM

Introduction

We know that there are many methods to analyze the collision velocity between vehicles; some methods are based on the retrieval data crash from the electronic device as well EDR. However, the use of physical models for the speed calculation solution is considered in this work.

About the set of physical equations, it can be used backward or forward to collision analysis, and its use can simplify algorithm creations. It represents the set of actions or step by step of calculations chain from equations used to find the solution.

Today in Latin America, many experts use the physical models from evidence to analyze the energy changes in each collision phase, from the last vehicle displacement that ends in the vehicle's rest position to the initial phase of collision (backward method).

If we call ΔE_i to the change energy of the vehicle in step "i" and ΔE_{i-1} to the change energy in the accident back step, any error adding in the velocity analysis in the last step "i" will carry over and may amplify the error in the previous stage "i+1".

In opposition to this, the forward method allows checking whether the energy transfer conditions of each stage are correct by comparing them with the evidence (move distance vehicle off step).

In the backward method, the input array – a set of inferred parameters and dates retrieval evidence, searches for the solution of delta-V. In contrast, the forward method input array – a set of inferred parameters or data retrieval evidence + hypothetical velocity- searches to calculate the distance of the post-collision displacement vehicle to check if the actual final position is the same as the calculated one.

It is clear that both arrays are differentiated by the "hypothetical speed" variable that the expert must propose, and this is not an easy task if you want to perform the analysis manually.

The forward method can only be used with the help of the computer, which through software can help us find the hypothetical speed values that, in combination with the other data in the input matrix, find the correct displacement value according to the evidence.

MERNG is the proposal to search for the values of the input matrix through the direct method that allows us to find the vehicle displacement result whose difference with the value of the evidence is minimal.

Method

The equation 1 and 2 are the typical expression of the Conservation of Lineal Momentum (CLM) to find the vehicles impact velocities by backward method. Normally that expression is used in perpendicular collision whit mass of the vehicles similarly.

$$m1 \cdot v1 \cdot \cos(\alpha 1) + m2 \cdot v2 \cdot \cos(\alpha 2) = m1 \cdot v1' \cdot \cos(\beta 1) + m2 \cdot v2' \cdot \cos(\beta 2) \qquad \text{Equation 1}$$

$$m1 \cdot v1 \cdot \sin(\alpha 1) + m2 \cdot v2 \cdot \sin(\alpha 2) = m1 \cdot v1' \cdot \sin(\beta 1) + m2 \cdot v2' \cdot \sin(\beta 2) \qquad \text{Equation 2}$$

Where:

m1, m2: are the vehicles mass #1 and #2 respectively in [kg].

v1´, v2´: are post-impact velocities vehicles #1 y #2 respectively in [m/sec].

α1, α2: are approach angle vehicles #1 y #2 in [degree].

β1, β2: are departure angle vehicles #1 y #2 in [degree].

The post-impact velocities equation can be introduced into in equations 1 and 2; always after the collision, both vehicles move without slope, and the friction or factor deceleration is known.

Solving the equations 1 and 2 to know the skid mark after the collision to position reset, we have:

$$d1 = \left(\frac{m1 \cdot v1 \cdot \cos(\alpha 1) + m2 \cdot v2 \cdot \cos(\alpha 2) - m2 \cdot \sqrt{c \cdot d2 \cdot \mu 2} \cdot \cos(\beta 2)}{m1 \cdot \cos(\beta 1)} \right)^2 \cdot \frac{1}{c \cdot \mu 1} \qquad \text{Equation 3}$$

$$d2 = \left(\frac{m1 \cdot v1 \cdot \sin(\alpha 1) + m2 \cdot v2 \cdot \sin(\alpha 2) - m1 \cdot \sqrt{c \cdot d1 \cdot \mu 1} \cdot \sin(\beta 1)}{m2 \cdot \sin(\beta 2)} \right)^2 \cdot \frac{1}{c \cdot \mu 2} \qquad \text{Equation 4}$$

Where:

μ1, μ2: are the friction coefficient of drag factor after Collision without units.

d1, d2: are module off displace vector #1 y #2 respectively, from collision to rest position in [m].

c: is a constant = 2.g, where g es gravitational acceleration = 9,81.m/sec^2.

The solution of the equations system 3 and 4 is not easy to solve; however, the proposal is to replace d1 in equation 4 and d2 in equation 3 and solve both. But will be necessary the use of term groupings will be necessary. In this way, it can be grouped as follows:

$$A_1 = m1 \cdot v1 \cdot \cos(\alpha 1) + m2 \cdot v2 \cdot \cos(\alpha 2)$$

$$B_1 = m2 \cdot \cos(\beta 2)$$

$$C_1 = m1 \cdot \cos(\beta 1)$$

$$D_1 = m1 \cdot v1 \cdot \sin(\alpha 1) + m2 \cdot v2 \cdot \sin(\alpha 2)$$

$$E_1 = m1 \cdot \sin(\beta 1)$$

$$F_1 = m2 \cdot \sin(\beta 2)$$

$$G_1 = c \cdot \mu 1$$

The coefficients from A_1 to G_1 were developed from set equation 3 when the distance "d2" of equation 4 is replaced in the 3. And the coefficients from A_2 to G_2 were developed from set equation 4 when the distance "d1" of equation 3 is replaced in the 4.

$$A_2 = m1 \cdot v1 \cdot \sin(\alpha 1) + m2 \cdot v2 \cdot \sin(\alpha 2)$$

$$B_2 = m1 \cdot \sin(\beta 1)$$

$$C_2 = m2 \cdot \sin(\beta 2)$$

$$D_2 = m1 \cdot v1 \cdot \cos(\alpha 1) + m2 \cdot v2 \cdot \cos(\alpha 2)$$

$$E_2 = m2 \cdot \cos(\beta 2)$$

$$F_2 = m1 \cdot \cos(\beta 1)$$

$$G_2 = c \cdot \mu 2$$

The solution of equation system from 3 and 4 can it solve consider the set equation below, and the end expression is:

$$d1 = \frac{(A_1)^2 \cdot (F_1)^2 - 2 \cdot A_1 \cdot B_1 \cdot D_1 \cdot F_1 + (B_1)^2 \cdot (D_1)^2}{G_1 \cdot (B_1)^2 \cdot (E_1)^2 - 2 \cdot G_1 \cdot B_1 \cdot C_1 \cdot E_1 \cdot F_1 + G_1 \cdot (C_1)^2 \cdot (F_1)^2} \qquad \text{Equation 5}$$

$$d2 = \frac{(A_2)^2 \cdot (F_2)^2 - 2 \cdot A_2 \cdot B_2 \cdot D_2 \cdot F_2 + (B_2)^2 \cdot (D_2)^2}{G_2 \cdot (B_2)^2 \cdot (E_2)^2 - 2 \cdot G_2 \cdot B_2 \cdot C_2 \cdot E_2 \cdot F_2 + G_2 \cdot (C_2)^2 \cdot (F_2)^2}$$ Equation 6

Finally, if we call D1 and D2 as modulus of the distance traveled by vehicles #1 and #2, respectively, in meters, from the beginning of the separation to their final position; and the values of the input variables are correct, we will have:

$d1 - D_1 = 0$ and $d2 - D_2 = 0$

We will call this difference “error”, and we will search by means of the MERNG method that the error will be zero or minimum. The input array and output with 16 variables are defined in Excel® as well as in the Figure 1.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Vel impact #1	Mass #1	Approach #1	Departure #1	Distance #1	Friction #1	**Vel impact #2**	Mass #2	Approach #2	Departure #2	Distance #2	Friction #2	d1 calculated #1	d2 calculated #2	**Error #1**	**Error #2**
[km/hr]	[kg]	[degree]	[degree]	[m]	no unit	**[km/hr]**	[kg]	[degree]	[degree]	[m]	no unit	[m]	[m]	[m]	[m]

FIGURE 1: MERNG INPUT AND OUTPUT ARRAY.

The values D1 and D2, which are determined by the evidence, are defined in columns E” and “K”. On the other hand, the verification that the input values are correct is performed in the “O” and “P” columns O” and “P”. Finally, the equations of the set developed from equation 3 and 4 will write in other columns of Excel®.

The Excel® work environment has a low application cost, and this software also contains a tool called the RANDOM BETWEEN function, which facilitates the generation of random numbers between extremes of each variable. Therefore, the entries in columns A, B, C, D, F, G, H, I, J and L, must be defined between a minimum and maximum hypothesis value according to the following Excel® rule:

=RANDBETEEN(minimum value; maximum value)

The values of columns O and P should compare as absolute values of the subtraction result between values calculated from columns M and N and total distance evidence entered in columns E and K (values D1 and D2). Figure 2.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Vel impact #1	Mass #1	Approach #1	Departure #1	Distance #1	Friction #1	**Vel impact #2**	Mass #2	Approach #2	Departure #2	Distance #2	Friction #2	d1 calculated #1	d2 calculated #2	Error #1	Error #2
[km/hr]	[kg]	[degree]	[degree]	[m]	no unit	**[km/hr]**	[kg]	[degree]	[degree]	[m]	no unit	[m]	[m]	[m]	[m]
60	1240,00	2,00	22,00	8,97	0,61	**24,00**	1420,00	85,00	33,00	2,52	0,58	15,13400893	2,125109887	**6,164009**	**0,39489**
53	1240,00	2,00	18,00	8,97	0,68	**15,00**	1420,00	89,00	28,00	2,52	0,70	12,92792554	0,389918243	**3,957926**	**2,130082**
49	1240,00	0,00	20,00	8,97	0,62	**25,00**	1420,00	87,00	30,00	2,52	0,70	0,050440179	14,76542325	**8,91956**	**12,24542**
54	1240,00	2,00	21,00	8,97	0,55	**23,00**	1420,00	87,00	27,00	2,52	0,70	8,49671E-05	20,96067588	**8,969915**	**18,44068**
45	1240,00	1,00	20,00	8,97	0,51	**26,00**	1420,00	88,00	32,00	2,52	0,66	0,405243964	22,83224989	**8,564756**	**20,31225**
56	1240,00	0,00	19,00	8,97	0,55	**14,00**	1420,00	90,00	29,00	2,52	0,64	40,86370155	1,707615048	**31,8937**	**0,812385**
54	1240,00	2,00	21,00	8,97	0,68	**10,00**	1420,00	90,00	28,00	2,52	0,52	71,60990699	14,09492462	**62,63991**	**11,57492**
53	1240,00	1,00	20,00	8,97	0,60	**30,00**	1420,00	87,00	28,00	2,52	0,63	9,817997896	53,27638897	**0,847998**	**50,75639**
43	1240,00	2,00	22,00	8,97	0,56	**16,00**	1420,00	85,00	35,00	2,52	0,63	12,21990584	0,277278812	**3,249906**	**2,242721**
55	1240,00	1,00	19,00	8,97	0,51	**20,00**	1420,00	89,00	25,00	2,52	0,59	2,249293848	11,02461727	**6,720706**	**8,504617**
54	1240,00	2,00	21,00	8,97	0,70	**17,00**	1420,00	90,00	34,00	2,52	0,61	17,2815669	0,029874066	**8,311567**	**2,490126**

FIGURE 2: MERNG TO 10 ITERATIONS.

It is important to remember that the "RANDBETEEN" function generates random values between the indicated extremes without privileging any probability distribution. Finally, after performing the 1000 simulations, the values of the "Error" columns can be filtered to find the smallest error between the results calculated by the direct method and the distance of the evidence with which it is compared.

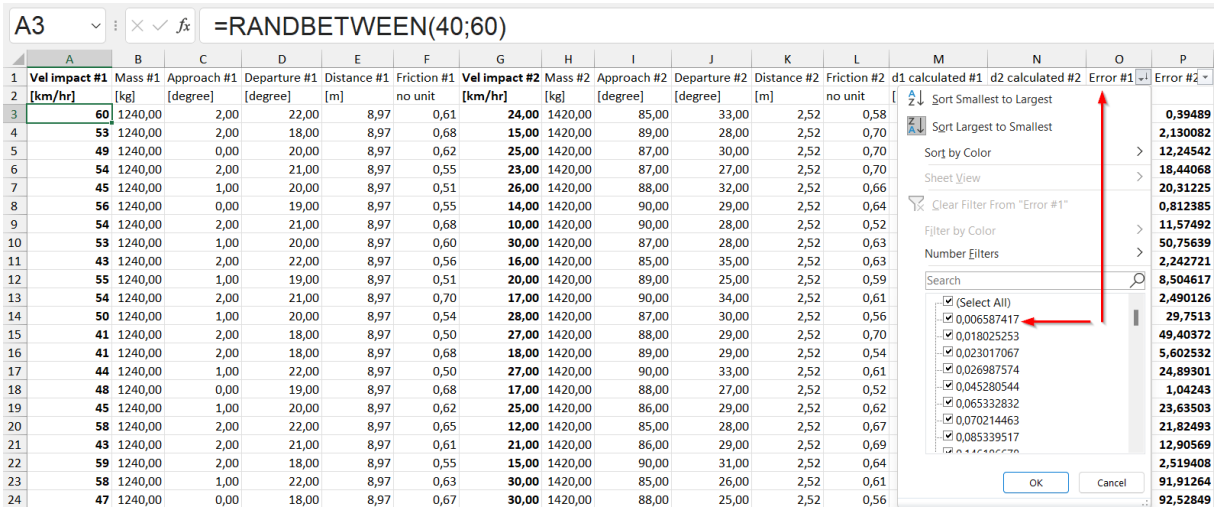
Results

To consider the MERNG proposal, a perpendicular collision (90°) between two vehicles of similar masses is analyzed (Chevrolet Aveo 1.6 #1 driving from right to left; Fiat Bravo 1.8 #2 driving from bottom to top). The initial point of contact is at the right front end of vehicle #1 and the left front end of vehicle #2. The information collected from the accident investigation is detailed below in Table 1:

Vehicle #1: Chevrolet Aveo 1.6	Vehicle #2: Fiat Bravo
Mass with occupants: 1240 kg.	Mass with occupants: 1420 kg.
Post-impact friction coefficient: between 0.5 - 0.7	Post-impact friction coefficient: between 0.5 - 0.7
Approach angle: between 0° and 5°	Approach angle: between 85° and 95°
Departure angle: between 18° and 22°.	Departure angle: between 25° and 35°
Post-impact CM displacement: 8.97 m.	Post-impact CG displacement: 2.52 m.

TABLE 1: DATAS VEHICLES CONSIDERED

Figure 3 shows a part of the 1000 simulations generated and how the iterations with the least error are filtered.



	Vel impact #1	Mass #1	Approach #1	Departure #1	Distance #1	Friction #1	Vel impact #2	Mass #2	Approach #2	Departure #2	Distance #2	Friction #2	d1 calculated #1	d2 calculated #2	Error #1	Error #2
1	[km/hr]	[kg]	[degree]	[degree]	[m]	no unit	[km/hr]	[kg]	[degree]	[degree]	[m]	no unit				
3	**60**	1240,00	2,00	22,00	8,97	0,61	**24,00**	1420,00	85,00	33,00	2,52	0,58				**0,39489**
4	**53**	1240,00	2,00	18,00	8,97	0,68	**15,00**	1420,00	89,00	28,00	2,52	0,70				**2,130082**
5	**49**	1240,00	0,00	20,00	8,97	0,62	**25,00**	1420,00	87,00	30,00	2,52	0,70				**12,24542**
6	**54**	1240,00	2,00	21,00	8,97	0,55	**23,00**	1420,00	87,00	27,00	2,52	0,70				**18,44068**
7	**45**	1240,00	1,00	20,00	8,97	0,51	**26,00**	1420,00	88,00	32,00	2,52	0,66				**20,31225**
8	**56**	1240,00	0,00	19,00	8,97	0,55	**14,00**	1420,00	90,00	29,00	2,52	0,64				**0,812385**
9	**54**	1240,00	2,00	21,00	8,97	0,68	**10,00**	1420,00	90,00	28,00	2,52	0,52				**11,57492**
10	**53**	1240,00	1,00	20,00	8,97	0,60	**30,00**	1420,00	87,00	28,00	2,52	0,63				**50,75639**
11	**43**	1240,00	2,00	22,00	8,97	0,56	**16,00**	1420,00	85,00	35,00	2,52	0,63				**2,242721**
12	**55**	1240,00	1,00	19,00	8,97	0,51	**20,00**	1420,00	89,00	25,00	2,52	0,59				**8,504617**
13	**54**	1240,00	2,00	21,00	8,97	0,70	**17,00**	1420,00	90,00	34,00	2,52	0,61				**2,490126**
14	**50**	1240,00	1,00	20,00	8,97	0,54	**28,00**	1420,00	87,00	30,00	2,52	0,56				**29,7513**
15	**41**	1240,00	2,00	18,00	8,97	0,50	**27,00**	1420,00	88,00	29,00	2,52	0,70				**49,40372**
16	**41**	1240,00	2,00	18,00	8,97	0,68	**18,00**	1420,00	89,00	29,00	2,52	0,54				**5,602532**
17	**44**	1240,00	1,00	22,00	8,97	0,50	**27,00**	1420,00	90,00	33,00	2,52	0,61				**24,89301**
18	**48**	1240,00	0,00	19,00	8,97	0,68	**17,00**	1420,00	88,00	27,00	2,52	0,52				**1,04243**
19	**45**	1240,00	1,00	20,00	8,97	0,62	**25,00**	1420,00	86,00	29,00	2,52	0,62				**23,63503**
20	**58**	1240,00	2,00	22,00	8,97	0,65	**12,00**	1420,00	85,00	28,00	2,52	0,67				**21,82493**
21	**43**	1240,00	2,00	21,00	8,97	0,61	**21,00**	1420,00	86,00	29,00	2,52	0,69				**12,90569**
22	**59**	1240,00	2,00	18,00	8,97	0,55	**15,00**	1420,00	90,00	31,00	2,52	0,64				**2,519408**
23	**58**	1240,00	1,00	22,00	8,97	0,63	**30,00**	1420,00	85,00	26,00	2,52	0,61				**91,91264**
24	**47**	1240,00	0,00	18,00	8,97	0,67	**30,00**	1420,00	88,00	25,00	2,52	0,56				**92,52849**

FIGURE 3: SEARCH FOR THE SMALLEST ERROR IN THE SIMULATION.

By filtering the minor error #1 and error #2, the following solution data is obtained, which is shown in Table 2:

INPUT VALUES		
Vehicle	#1 Chevrolet Aveo 1.6	#2 Fiat Bravo
Mass + driver	1240 kg.	1420 kg
Approach	1°	89°
Speed	**50.0**	**20.0**
OUTPUT VALUES		
Departure	21°	35°
Distance	8.96 m (error 0 m)	2.52 m (error 0.77 m)
Average friction coefficient	0.59	0.5

TABLE 2: INPUT AND OUTPUT VALUES ACQUIRED FROM MERNG.

Discussion

The results obtained from Excel® by this method showed are compared with optimizer collision solutions from Pc-crash®. The Figure 4 shows the PC-crash® workspace to search the collision solution from the same vehicles showed in the Table 1. The background used is the same accident diagram where it shows the rest position vehicles.

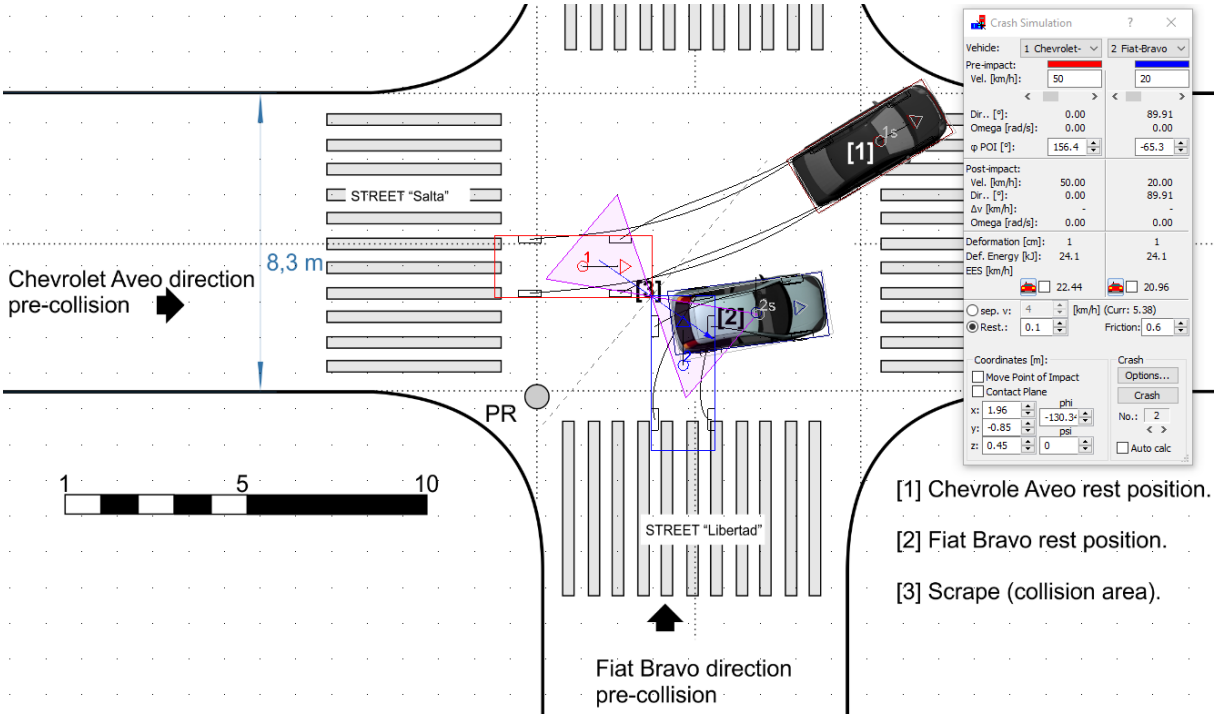


FIGURE 4: WORKSPACE PC-CRASH® WITH COLLISION SOLUTION.

Next, it's detailed the more important results download of PC-Crash® report. Table 3.

Vehicle	Chevrolet-Aveo 1.6 -	Fiat-Bravo 1.8 Flex -
RecordID:	72297	67741
STAR VALUES		
Velocity magnitude (v) [km/h]:	**50.00**	**20.00**
Total weight [kg]:	1240.00	1420.00
Center of gravity x [m]:	0.04	2.85
Center of gravity y [m]:	-0.01	-2.78
Velocity direction (ß) [deg]:	0.00	89.91
END VALUES		
Center of gravity x [m]:	8.30	4.89
Center of gravity y [m]:	3.49	-1.30
Velocity direction (ß) [deg]:	20.47	31.55
Specified brake acceleration [m/s²]:	-4.91	-5.8
Friction coefficient	0.8	0.8

TABLE 3: DETAILED OF THE MORE IMPORTANT RESULTS DOWNLOAD OF PC-CRASH® REPORT.

Although the defined input and output angles values between MERNG and PC-Crash® are similar and validated by rest positions evidence, other variables don't match as the friction coefficient after a collision.

The collision solution proposed by PC-Crash® results in a more complete model [6], than the one proposed by equations 3 and 4, which corresponds to the simple Conservations of Lineal Momentum analysis for particles.

Variables, as well as angular momentum, and inertial moment, all from planar mechanics, are not considered in this example. Also, this method doesn't offer any solution to show graph verification, as is PC-crash®.

However, MERNG has the benefit of its low cost to be used and could be helpful in developed the simple collision models, although the expert needs to know low code programming, some as example Excel®.

Conclusion

The proposed MERNG method is based on constructing an algorithm where the expert can control multiple input values and search by comparison (difference) with the evidence of post-impact displacement, the correct data matrix that validates the analysis solution.

The use of low code to create calculation algorithms can be developed in environments such as Excel®.

This method could also be a preparation tool for many experts in Latin America who, until today, continue to solve speed analysis using the backward method in order to understand the benefits of the forward method.

The method has the following disadvantage: it is indicated the lack of a solution graphic display, the need for the expert to know how to build for each type of collision, the algorithm necessary to search for the solution, and to depend on his ability to represent the algorithm of the solution in a platform where they can run the simulation.

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