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ABSTRACT

In the investigation of a catastrophe it is important the determination of the speeds of the units interveners. Therefore, generally, the reconstruction of sinister are made from a model based in the variables values measured in the sinister scene. In this work, the influence of uncertainties involved in restrained distances is investigated and the choice of a friction coefficient value, μ, is made to determinate the circulation velocity. As a conclusion, the uncertainties cause an important error propagation, which increases with decreasing values of μ, making the situation more critical when the estimated velocity value is around 11,11m/sec or (24,85mph), which has a critical understanding at the judicial area, for our regulation of the traffic (in Argentina).

INTRODUCTION

The veracity of conclusions at the expert report depends on the fact if the used procedure to get evidences at the fact place presents some kind of error o errors, it belongs to systematic errors or hazard errors. They will also affect shunt magnitudes because of the propagations. This is an important problem in accident reconstruction cases, in which ones it is necessary to take account of a lot of variables . In the particular case where a mobile unit's velocity is calculated as a function of kinetic energy losses, because of a friction work, the variables are: kinetic friction coefficient and measured restrained distance. The procedure has two parts: 1)data collection and 2)calculation work. The measurements of magnitudes and the correspondents uncertainty determination are made at the fact place, in data collection work.

The Prints are, generally, rubber pieces on the bearing surface (asphalt), that takes a segment form with an specific wide and different prolongations. On these evidences, the rubber deposition is not homogeneous and this fact causes a difficulty in the visual appreciation of the print. This difficulty affects the interpretation of its beginning and ending points. The impossibility of making a certain estimation of the initial point of the print, and eventually, its final point, causes errors in the print length measurements. It is important to remark that the obtained distance measurement must be corrected in the model through the time reaction driver consideration.

An uncertainty source is originated in friction coefficient values, μ, and this is because of the different designs and materials in the manufacture of tires and kind of circulation surfaces.

MATERIALS AND METHOD

We will analyze different aspects related with error propagation applied to our case, in the detention of a vehicle which velocity is known and with specific conditions. In this experiment, the velocity of the vehicle is 40km/hr (24,85mph), because of the importance that this value has in the legal point of.

- Vehicle: Motorbike Honda 90cm^3
- Velocity of circulation: 40km/hr = 11,11m/sec (24,85mph)
- Average restrained distance : 10,1m
- Surface: armed concrete, dry and clean
- Average friction coefficient : μ = 0.6 (defined by bibliography)
- Slope of the road: 0%

ERROR CONTRIBUTION IN THE CALCULATION OF VELOCITY AS A FUNCTION DEPENDENT OF THE FRICTION COEFFICIENT
References:

- $s_{(\mu)}$: sensibility or standard error (measure of absolute dispersion)
- $\delta_{(\mu)}$: deviation of a value from the average

- μ : Average friction coefficient value
- d : Average restrained distance in meters
- v: velocity of circulation used in the experiment measured in m/sec
- s(v): the error contributed for the circulation speed in dispersion units
- c: double value of acceleration of gravity (m/sec^2)
- Cv: variation coefficient in %(measure of relative dispersion)

Proposed equation for the velocity calculation using kinetic energy loss, [Sears, Zemansky, Young, 1988]:

$$v = \sqrt{c.\mu..d} \qquad 1$$

First, let's consider the velocity deviation from the friction coefficient, when the operator is a tenth far from the average value of μ.

Value averages:
v = 11,11 m/sec d = 10,2 m μ = 0,6
Doble value of gravity acceleration: c =19,62 m/sec^2

$\sum_1^2 \delta(\mu) = 0.2$ sums of the deviation of the coefficient for μ = 0.5 and μ = 0.7

$$s(\mu) = \sqrt{\frac{(0.6-0.5)^2-(0.6-0.7)^2}{2}} = 0.1 \qquad 2$$

$$Cv = \frac{s(\mu)}{\mu} = 16.67\% \qquad 3$$

The Equation 2 defines the error of the friction coefficient (μ) for the treated cases, when it exists a separation from the central value of μ in a tenth difference. The error contribution in the circulation velocity, according to the proposed Equation 1 and as a function dependent of the deviation of μ , is defined by the following expression, [Brees 1975], [Galloni 1982], [Sixtos 1957], [Toranzos 1971]:

$$s(v) = \sqrt{\left(\frac{\partial v}{\partial \mu}\right)^2 .s(\mu)^2} \qquad 4$$

solving those partial derivate

$$s(v) = \sqrt{\left(\frac{c^{1/2}.d^{1/2}}{2.\sqrt{\mu}}\right)^2 .s(\mu)^2} \qquad 5$$

simplifying

$$s(v) = \sqrt{\left(\frac{c.d}{4.\mu}\right).s(\mu)^2} \qquad 6$$

$$Cv = \frac{s(v)}{v} = 8.19\% \qquad 7$$

The Equation 6 will allow us to appreciate the error contribution because of the deviation of μ. For our experiment, the appreciated error in the velocity calculation is:

s(v) = 0.9 error propagated in the speed, Ecuation 6

Cv = 8.19% error propagated in the speed, Ecuation 7

The following Table 1, expresses the different values of the error in the velocity calculations, taken from other central values of μ, with the same sensibility (μ) = 0.1, using Equation 6:

TABLE 1. Value of the error in the speed calculated for different central values of μ (fricction coefficient).

Value average of μ	Error propagated in the speed, s(v)	Variation Coefficient
0.9	0.75	6.75
0.8	0.79	7.11
0.7	0.85	7.65
0.6	0.9	8.2
0.5	1	9
0.4	1.12	10.08
0.3	1.29	11.61
0.2	1.58	14.2
0.1	2.24	20.1

The Figure 1 is the correspondent graphic the Table 1.

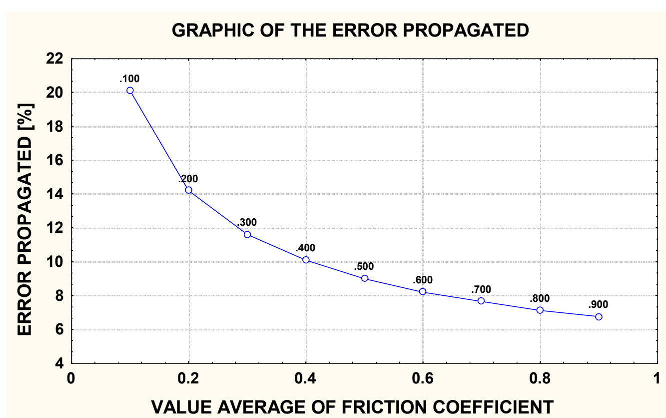

Figure 1. Graphic of the error propagated in funtion of value average of fricction coefficient.

ERROR CONTRIBUTION IN THE CALCULATION OF VELOCITY AS A FUNCTION DEPENDENT OF THE TRACE LENGTH

Now , let's consider the deviation of the velocity value from the dispersion of the different trace length measurements.

References:

- a, b, c, d: measures of trace length in meters
- D: average trace length value in meters
- δ(D): deviation of a value from the average
- s(D): sensibility or standard error (measure of absolute dispersion)
- s(v): the error contributed for the circulation speed in dispersion units
- Cv: variation coefficient in %(measure of relative dispersion)

a = 9.70 b = 10 c = 10.3 d = 10.5; D = 10.1

$$\delta(D) = (a-D)^2 + (b-D)^2 + (c-D)^2 + (d-D)^2$$

$$\delta(D) = 0.37$$

$$s(D) = \sqrt{\frac{\delta(D)}{3}} = 0.35 \quad (8)$$

$$Cv = \frac{\delta(D)}{D} = 3.48\% \quad (9)$$

The Equation 8, defines the standard error for the different measurements that were done on the restrained trace length. As the same as before, the error contribution in the circulation velocity calculation, as a function of the deviation s(D), will be defined with the following equation:

$$s(v) = \sqrt{\left(\frac{\partial v}{\partial D}\right)^2 .s(D)^2} \quad (10)$$

solving those partial derivate

$$s(v) = \sqrt{\left(\frac{c^{1/2}.\mu^{1/2}}{2.\sqrt{D}}\right)^2 .s(D)^2} \quad (11)$$

simplifying

$$s(v) = \sqrt{\left(\frac{c.\mu}{4.D}\right) .s(D)^2} \quad (12)$$

$$Cv = \frac{s(v)}{v} = 1.7\% \quad (13)$$

The Equation 12 will allow us to appreciate the error contribution because of the deviation of "D". For our experiment, it will be:

s(v) = 0.19 error propagated in the speed, Equation 12

Cv = 1.7% error propagated in the speed, Equation 13

Again, the following Table 2, gives the different values for the error calculations in the velocity values, taken from other central values of μ, with the same sensibility s(D) = 0.35, using Equation 8.

TABLE 2. Value of the error in the speed calculated for different central values of D (trace lengh).

Value average of trace lengh	Error propagated in the speed, s(v)	Variation Coefficient
10.1	0.19	1.7
8	0.21	1.9
7	0.23	2.1
6	0.25	2.3
5	0.27	2.4
4	0.3	2.7

Now, let's compute all the calculated contributions, the appreciated velocity error, from the deviation of μ and the restrained trace length. Then, the equation will be:

$$s(v) = \sqrt{\left(\frac{\partial v}{\partial \mu}\right)^2 .s(\mu)^2 + \left(\frac{\partial v}{\partial D}\right)^2 .s(D)^2} \quad (14)$$

solving

$$s(v) = \sqrt{\left(\frac{c.D}{4.\sqrt{\mu}}\right) .s(\mu)^2 + \left(\frac{c.\mu}{4.\sqrt{D}}\right) .s(D)^2} \quad (15)$$

$$Cv = \frac{s(v)}{v} = 8.2\% \quad (16)$$

Computing the values, finally we obtain the contribution of both deviations, friction coefficient and restrained trace length, for the appreciated error in the velocity calculation on the vehicle used in the experiment:

s(v) = 0.91 error propagated in the speed, Equation 15

Cv = 8.2% error propagated in the speed, Equation 16

The last obtained value in Equation 15, (0,91) is because of the sensibility or standard deviation of the velocity which are related with the appreciated errors in the restrained trace length and friction coefficient value; they were treated as independent error sources. In this way, and as we can appreciate in the previous equations, the correlation coefficient is considered zero.

DISCUSSION OF RESULTS

According to the estimated results , when a value of μ is taken from a table and it only has a difference of a tenth from the average value, which corresponds to an error of 16,6% , it will have an error in the estimation of velocity, and it will be higher when the average value of μ is smaller. In the Equation 2 it shows that for any central value of μ in the pointed conditions, the error will be always the same, $s(\mu) = 0{,}1$, but according to the Equation 6, and related with the same variable μ, it verifies that for smaller central values of the friction coefficient, the error in the velocity will be higher.

Note the different measurements that several operators done on the length of the restrain trace, which has a discrepancy around a 3,5% according to the deviation $s(D) = 0{,}35$, Equation 8. This error represents an error in the velocity around a $s(v) = 0.19$, Equation 12, which corresponds to an propagated error of 1,7%, Equation 13, but the error in the velocity that only considers the friction coefficient is 0.9, Equiation 6, equivalent to 8,1%, Equation 7.This last pointed error is the error in the velocity value as a function of the error of μ error, is more important that the first one (error of the velocity as a function of the error that has the distance).

With both errors, in the friction coefficient and restrain trace length, the propagation of error on the velocity final value is $s(v) = 0.91$, Equation 15, equivalent to 8,2%, Equiation 16.

CONCLUSION

The previous analysis shows that the initial Equation 1 is a propitious situation for error multiplication, in the calculations of a sinister analysis.

First, the risk in the error production, is not only a fact that depends of the average friction coefficient used in the calculations, but also it is important to know that when the average friction coefficient is smaller (μ), the value of the error in the appreciated velocity will be higher. In this sense, it seems that this fact answers to the initial written equation, where the velocity is not a linear function of μ, and the curve is not uniform; it presents increments on any point of its slope, when the coefficient values are near the origin (they tend to zero).

On the other way, according to the considerations and calculations related with the deviations of the restrained trace length and its correlation with the final velocity error, we must point that this error takes a secondary place compared with the another variable, (μ) when error propagation is calculated.

As an answer to the researcher's aspirations, the appreciated errors in the velocity values are tolerable, when they are treated without restrained trace length and friction coefficient considerations. However, when these variables are used in the initial equation used at the beginning of this study, the error propagation could take importance. This fact increases when the value of the involved velocity is around 11,11m/sec (40km/hr) or (2,85mph), which has a critical interpretation in the legal treatment, for our regulation of the traffic.

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